UCRHEP-T391
June 2005

Hiding the Existence of a Family Symmetry in the Standard Model

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Abstract

If a family symmetry exists for the quarks and leptons, the Higgs sector is expected to be enlarged to be able to support the transformation properties of this symmetry. There are however three possible generic ways (at tree level) of hiding this symmetry in the context of the Standard Model with just one Higgs doublet. All three mechanisms have their natural realizations in the unification symmetry E_6 and one in $SO(10)$. An interesting example based on $SO(10) \times A_4$ for the neutrino mass matrix is discussed.

There are three families of quarks and leptons. The pattern of their masses and mixing angles has been under study for a long time. If a family symmetry exists at the Lagrangian level, broken presumably only spontaneously and by explicit soft terms, the Yukawa couplings $f_{ijk} q_i q_j^c \phi_k$ and $f'_{ijk} l_i l_j^c \phi_k$ should have two or more Higgs doublets ϕ_k. Otherwise f_{ijk} and f'_{ijk} would reduce to f_{ij} and f'_{ij}. Since the number of bilinear invariants of any given symmetry is very much limited, this would not result in a realistic description of quark and lepton mass matrices. On the other hand, the well-tested Standard Model (SM) requires only one Higgs doublet (although it remains to be discovered experimentally). One Higgs doublet is also preferred phenomenologically as an explanation of the natural suppression of flavor-changing neutral currents [1]. Thus an important theoretical question is whether a family symmetry can be hidden in the context of the SM and how. The answer is yes and there are three generic mechanisms (at tree level) for achieving it, as shown below. Specific new particles with masses well above the electroweak scale are required, but some of these exist already in well-known unification symmetries such as E_6 and $SO(10)$.

The idea is very simple. The information concerning the family symmetry is encoded in the Yukawa couplings f_{ijk} of quarks through the various ϕ_k Higgs doublets. If only one Higgs doublet is allowed, the same information can be encoded using the dimension-five operator [2]

$$\mathcal{L}_Y = \frac{f_{ijk}}{\Lambda} q_i q_j^c \phi \sigma_k + H.c., \tag{1}$$

where σ_k are heavy scalar singlets. This mechanism is widely used in model building but without any discussion of how it may arise from fundamental interactions. Of course, if the new interactions occur near the Planck scale, then they may be very strong and the effective operator of Eq. (1) is nonperturbative in general. However, if the new physics responsible for the family symmetry is at or below the quark-lepton unification scale of about 10^{16} GeV, then it is reasonable to ask how it may be realized at tree level. In the following it is shown

that there are three generic ways of doing this, and each will be discussed also in the context of the complete underlying new physics involved. The assumption of this operator means that one or more of these generic mechanisms is likely to be correct and is thus an important clue to physics beyond the SM.

Even a casual observation of Eq. (1) shows that the four fields involved can be grouped into the product of two pairs in only three ways, in exact analogy to the classic analysis of the scattering of two particles into two particles. The intermediate states must then have well-defined transformation properties under the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. These are depicted in Figures 1 to 3. The heavy quarks $Q_{1,2}$ and $Q^c_{1,2}$ are $SU(2)_L$ singlets (doublets) and H are heavy scalar doublets.

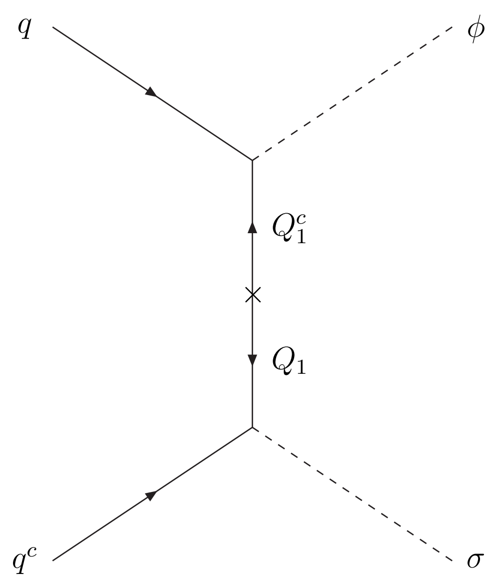

Figure 1: Realization of Eq. (1) with heavy Q_1 and Q_1^c singlets.

Consider first Fig. 1. Under $SU(3)_C \times SU(2)_L \times U(1)_Y$, $q \sim (3, 2, 1/6)$ and $\phi \sim (1, 2, \pm 1/2)$, hence $Q_1^c \sim (3^*, 1, -2/3)$ or $(3^*, 1, 1/3)$ is required. This means $Q_1 \sim (3, 1, 2/3)$ or $(3, 1, -1/3)$, i.e. heavy quark singlets with charges equal to either those of the u quarks

or d quarks. This mechanism is thus equivalent to that of the canonical seesaw mechanism for Dirac fermions [3]. The effective family structure of Eq. (1) is then given by

$$\frac{f_{ijk}}{\Lambda} = y_{ia}(M^{-1})_{ab}h_{bjk}, \tag{2}$$

where y_{ia} are the couplings of $q_i(Q_1^c)_a\phi$, h_{bjk} those of $(Q_1)_b q_j^c \sigma_k$, and M the mass matrix of $Q_1 Q_1^c$. Since the family symmetry applies to all three of these quantities, it is well hidden in the resulting effective operator of Eq. (1) and even more so in the resulting mass matrix

$$m_{ij} = \frac{f_{ijk}}{\Lambda}\langle\sigma_k\rangle\langle\phi\rangle. \tag{3}$$

On the positive side, if this particular mechanism is assumed, specific models of family structure may be considered and then compared to the data. Singlet quarks of charge $-1/3$ are contained in the fundamental $\underline{27}$ representation of E_6. Hence the d quarks of the SM may owe their family structure wholly or partly [4] to such a mechanism.

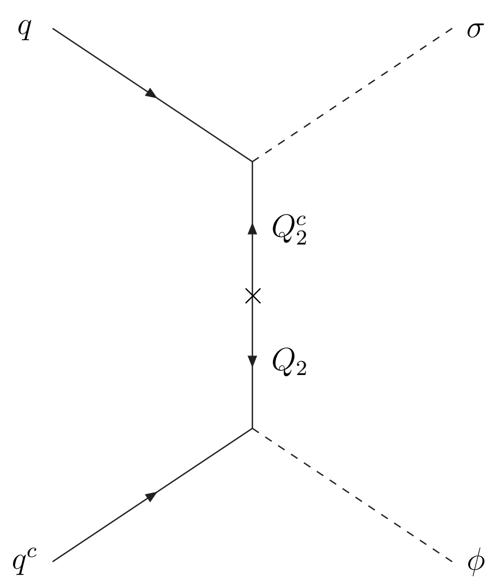

Figure 2: Realization of Eq. (1) with heavy Q_2 and Q_2^c doublets.

Consider next Fig. 2. Here $Q_2 \sim (3, 2, 1/6)$ and $Q_2^c \sim (3^*, 2, -1/6)$ are required. Whereas these heavy vector quark doublets are not present in the $\underline{27}$ of E_6, the corresponding heavy

lepton doublets $L_2 \sim (1,2,-1/2)$ and $L_2^c \sim (1,2,1/2)$ are, and they have been used for example in a recently proposed model [5] of late neutrino mass and baryogenesis. Thus the observed lepton family structure may be encoded with this mechanism.

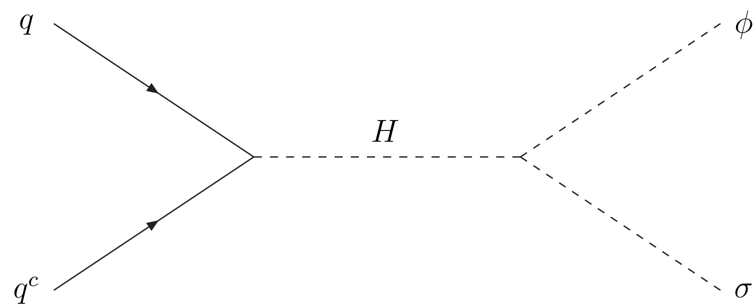

Figure 3: Realization of Eq. (1) with heavy scalar H doublets.

Finally consider Fig. 3. The analog of Eq. (2) is

$$\frac{f_{ijk}}{\Lambda} = h_{ija}(M^2)^{-1}_{ab}\mu_{bk}, \tag{4}$$

where h_{ija} are the couplings of $q_i q_j^c H_a$, μ_{bk} those of $H_b^\dagger \phi \sigma_k$, and M^2 the mass-squared matrix of H. This mechanism is realized naturally for example in $SO(10)$ (as well as E_6), where q and q^c are $SU(2)_L$ and $SU(2)_R$ doublets respectively, H the heavy scalar bidoublets which carry the family structure, and ϕ the SM scalar doublet. It is appplicable to all Dirac fermions, including the u quarks. It differs from the usual realization of quark masses in left-right gauge models where H is a scalar bidoublet at the electroweak scale.

Since qq^c couples to $\phi\sigma$ through H in Fig. 3, the full Higgs potential involving all 3 scalar fields should be considered. As a simple example, consider the case where σ and H transform in the same way under an extra $U(1)$ symmetry but ϕ is trivial, so that $H^\dagger\phi\sigma$ is an allowed term in the Lagrangian but $H^\dagger\phi$ is not. The most general Higgs potential involving σ, H, and ϕ is then given by

$$V = m_\sigma^2 \sigma^\dagger\sigma + m_H^2 H^\dagger H + m_\phi^2 \phi^\dagger\phi + \frac{1}{2}\lambda_1(\sigma^\dagger\sigma)^2 + \frac{1}{2}\lambda_2(H^\dagger H)^2 + \frac{1}{2}\lambda_3(\phi^\dagger\phi)^2$$

$$+ \quad \lambda_4(\sigma^\dagger\sigma)(H^\dagger H) + \lambda_5(\sigma^\dagger\sigma)(\phi^\dagger\phi) + \lambda_6(H^\dagger H)(\phi^\dagger\phi) + \lambda_7(H^\dagger\phi)(\phi^\dagger H)$$

$$+ \quad [\mu H^\dagger \phi\sigma + H.c.] \tag{5}$$

Let μ be real, as well as $\langle\sigma\rangle = x$, $\langle H\rangle = u$, and $\langle\phi\rangle = v$. Then the minimization of V results in the 3 conditions:

$$x[m_\sigma^2 + \lambda_1 x^2 + \lambda_4 u^2 + \lambda_5 v^2] + \mu uv = 0, \tag{6}$$

$$u[m_H^2 + \lambda_2 u^2 + \lambda_4 x^2 + (\lambda_6+\lambda_7)v^2] + \mu vx = 0, \tag{7}$$

$$v[m_\phi^2 + \lambda_3 v^2 + \lambda_5 x^2 + (\lambda_6+\lambda_7)u^2] + \mu ux = 0. \tag{8}$$

Since $u, v << x$ is required for electroweak symmetry breaking,

$$x^2 \simeq \frac{-m_\sigma^2}{\lambda_1} \tag{9}$$

is obtained from Eq. (6). Assuming now that $m_H^2 + \lambda_4 x^2 > 0$, Eq. (7) then yields

$$u \simeq \frac{-\mu vx}{m_H^2 + \lambda_4 x^2}. \tag{10}$$

Substituting the above into Eq. (8), the following effective condition for v is obtained:

$$m_\phi^2 + \lambda_5 x^2 - \frac{\mu^2 x^2}{m_H^2 + \lambda_4 x^2} + \left[\lambda_3 + \frac{(\lambda_6+\lambda_7)\mu^2 x^2}{(m_H^2 + \lambda_4 x^2)^2}\right] v^2 = 0. \tag{11}$$

Using Eqs. (6) to (8), the mass-squared matrix spanning the neutral real components of σ, H, and ϕ is given by

$$\mathcal{M}^2_{\sigma,H,\phi} = \begin{pmatrix} 2\lambda_1 x^2 - \mu uv/x & 2\lambda_4 xu + \mu v & 2\lambda_5 xv + \mu u \\ 2\lambda_4 xu + \mu v & 2\lambda_2 u^2 - \mu vx/u & 2(\lambda_6+\lambda_7)uv + \mu x \\ 2\lambda_5 xv + \mu u & 2(\lambda_6+\lambda_7)uv + \mu x & 2\lambda_3 v^2 - \mu ux/v \end{pmatrix}. \tag{12}$$

Since $x >> u, v$, two approximate eigenstates are σ and $(vH - u\phi)/\sqrt{v^2+u^2}$ with $m^2 \simeq 2\lambda_1 x^2$ and $-\mu x(v^2+u^2)/vu$ respectively. Thus all scalar fields are heavy except for the linear combination $(v\phi + uH)/\sqrt{v^2+u^2}$ which is identical to the single Higgs doublet of the SM. If the latter is extended to include supersymmetry, then there will be two Higgs doublets, as in the MSSM (Minimal Supersymmetric Standard Model).

In the mechanism of Fig. 3, it is clear that the qq^c mass matrix may also be written as

$$m_{ij} = h_{ijk}\langle H_k\rangle, \tag{13}$$

where $\langle H_k\rangle$ is given by the generalization of Eq. (10). The family structure is determined not only by h_{ijk} which may come from an assumed symmetry, but also by $\langle H_k\rangle$ which is hidden in the dynamics of the scalar sector much above the electroweak scale. However, if the family symmetry is global, and broken only spontaneously, then a massless Goldstone boson, the familon, will appear [6].

As an application of the mechanism of Fig. 3, consider the non-Abelian discrete symmetry A_4, the group of the even permutation of 4 objects which is also the symmetry group of the tetrahedron. It has been discussed [7, 8, 9, 10, 11] as a family symmetry for the understanding of the neutrino mass matrix. Suppose it is combined with $SO(10)$. Then all quarks and leptons are naturally assigned as $(\mathbf{16};\underline{3})$ under $SO(10)\times A_4$. [There are 3 inequivalent irreducible singlet representations of A_4, $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and 1 irreducible triplet representation $\underline{3}$.] This assignment differs from the original one [7, 8] where $q,l\sim\underline{3}$ but $q^c,l^c\sim\underline{1},\underline{1}',\underline{1}''$, which cannot be embedded into $SO(10)$. The heavy scalar H should then be assigned as $(\overline{\mathbf{10}};\underline{1},\underline{1}',\underline{1}'')$, σ as $(\overline{\mathbf{16}};\underline{1},\underline{1}',\underline{1}'')$, and ϕ as $(\overline{\mathbf{16}};\underline{1})$. For $a_{1,2,3}\sim\underline{3}$ and $b_{1,2,3}\sim\underline{3}$ under A_4,

$$a_1b_1 + a_2b_2 + a_3b_3 \sim \underline{1}, \tag{14}$$

$$a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3 \sim \underline{1}', \tag{15}$$

$$a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3 \sim \underline{1}'', \tag{16}$$

where $\omega = e^{2\pi i/3}$. Hence all quark and lepton Dirac mass matrices are diagonal but with arbitrary eigenvalues. This is actually a rather good approximation in the quark sector, where all mixing angles are known to be small. In fact, if the theory is also supersymmetric, then the explicit breaking of A_4 in the soft supersymmetry-breaking terms themselves could be used to generate a realistic quark mixing matrix [12]. In the lepton sector, the same could

be accomplished [13] in the case of the BMV model [8]. On the other hand, in the context of $SO(10)$, the $\overline{\mathbf{126}}$ representation can be used to obtain Majorana neutrino masses according to the well-known seesaw formula [14]

$$\mathcal{M}_\nu = \mathcal{M}_L - \mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T, \tag{17}$$

where

$$\mathcal{M}_D = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}. \tag{18}$$

As for $\mathcal{M}_{L,R}$, using the assignment $(\overline{\mathbf{126}}; \underline{3})$, they are both naturally of the form

$$\mathcal{M}_L = \begin{pmatrix} 0 & d & d \\ d & 0 & d \\ d & d & 0 \end{pmatrix}, \quad \mathcal{M}_R = \begin{pmatrix} 0 & D & D \\ D & 0 & D \\ D & D & 0 \end{pmatrix}. \tag{19}$$

Thus

$$\mathcal{M}_R^{-1} = \frac{1}{2D} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}, \tag{20}$$

and

$$\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T = \frac{1}{2D} \begin{pmatrix} -x^2 & xy & xz \\ xy & -y^2 & yz \\ xz & yz & -z^2 \end{pmatrix} = - \begin{pmatrix} a & b & c \\ b & b^2/a & -bc/a \\ c & -bc/a & c^2/a \end{pmatrix}, \tag{21}$$

which has 3 zero 2×2 subdeterminants as expected [15]. Together with $\mathcal{M}_L$, this becomes a four-parameter hybrid description [16] of the neutrino mass matrix. Let $b = c$, then in the basis spanning ν_e, $(\nu_\mu + \nu_\tau)/\sqrt{2}$, and $(-\nu_\mu + \nu_\tau)/\sqrt{2}$,

$$\mathcal{M}_\nu = \begin{pmatrix} a & \sqrt{2}(d+b) & 0 \\ \sqrt{2}(d+b) & d & 0 \\ 0 & 0 & -d + 2b^2/a \end{pmatrix}, \tag{22}$$

which is a new and very interesting pattern. It implies that $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ in the neutrino mixing matrix, in agreement with data. Assuming d, a, b to be real, the solar mixing angle is given by

$$\tan 2\theta_{12} = \frac{2\sqrt{2}(d+b)}{d-a}, \tag{23}$$

which yields $\tan^2\theta_{12} = 0.5$ in the limit $a = b = 0$, again in agreement with data and realizing the so-called tri-bimaximal mixing pattern suggested [17] some time ago. Assuming thus that

$$a << b << d << b^2/a, \tag{24}$$

the 3 neutrino mass eigenvalues in this case are approximately $-d$, $2d$, and $2b^2/a$, resulting in

$$\Delta m^2_{sol} \simeq 3d^2, \quad \Delta m^2_{atm} \simeq 4b^4/a^2. \tag{25}$$

As a numerical example, let

$$a = 2.6 \times 10^{-5} \text{ eV}, \quad b = -8.5 \times 10^{-4} \text{ eV}, \quad d = 5.5 \times 10^{-3} \text{ eV}, \tag{26}$$

then

$$\tan^2\theta_{12} = 0.45, \quad \Delta m^2_{sol} = 7.9 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{atm} = 2.4 \times 10^{-3} \text{ eV}^2. \tag{27}$$

More details of this model will be presented elsewhere.

In conclusion, it has been shown how the Standard Model with one Higgs doublet may hide the existence of a family symmetry for the quarks and leptons. The new heavy particles involved in 3 tree-level realizations of this mechanism have been identified and found to be available in the unification symmetries E_6 and $SO(10)$. A new and very interesting model based on $SO(10) \times A_4$ for the neutrino mass matrix is obtained.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

[1] S. L. Glashow and S. Weinberg, Phys. Rev. **D15**, 1958 (1977).

[2] C. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[3] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Phys. Rev. **D36**, 1479 (1987).

[4] J. L. Rosner, Phys. Rev. **D61**, 097303 (2000).

[5] L. J. Hall, H. Murayama, and G. Perez, hep-ph/0504248.

[6] F. Wilczek, Phys. Rev. Lett. **49**, 1549 (1982).

[7] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[8] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[9] E. Ma, Phys. Rev. **D70**, 031901(R) (2004).

[10] G. Altarelli and F. Feruglio, hep-ph/0504165.

[11] E. Ma, hep-ph/0505209.

[12] K. S. Babu, B. Dutta, and R. N. Mohapatra, Phys. Rev. **D60**, 095004 (1999).

[13] M. Hirsch, J. C. Romao, S. Skadhauge, J. W. F. Valle, and A. Villanova del MOral, Phys. Rev. **D69**, 093006 (2004).

[14] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba,

Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[15] E. Ma, hep-ph/0501056.

[16] This differs from the proposal of S.-L. Chen, M. Frigerio, and E. Ma, hep-ph/0504181, where $\mathcal{M}_L$ is proportional to the unit matrix.

[17] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530**, 167 (2002).